# Theory of Automata - HW2 

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## 1 Question 1

For $\sum=\{a, b\}$, construct dfa's that accept the sets consisting of

- (a) all string exactly one a,
- (d) all strings with at least one $a$ and exactly two $b$ 's.

Answers: (a) a Dfa that accepts at least one $a$. From initial states, once the Dfa read a string 'a' it will go to final state no matter which characters being read next. The dfa is shown in Fig. 1


Figure 1: A dfa accepts at least one $a$
Answers: (b) a dfa that accepts at least one $a$ and exactly two $b$ 's. The answer is shown in Fig. 2


Figure 2: A dfa accepts at least one $a$, and exactly two b

## 2 Question 3

Give a set of notation description of the language accepted by the automaton depicted in the following diagram. Can you think of a simple verbal characterization of the language?.

This dfa accepts any string that contains at least one $a$ and not ending with $a$.

## 3 Question 4

Find a dfa for the following language on $\sum=\{a, b\}$.
$L=\left\{w: n_{a}(w) \bmod 3>1\right\}$
The dfa is shown in Fig. 3


Figure 3: A Machine accepts all string with $\bmod 3>1$

## 4 Question 6

Design an nfa with no more than five states for the set $\left\{a b a b^{n}: n \geq 0\right\} \cup\left\{a b a^{n}: \geq 0\right\}$. The nfa is shown in Fig. 4


Figure 4: A nfa accepts the set $\left\{a b a b^{n}: n \geq 0\right\} \cup\left\{a b a^{n}: \geq 0\right\}$

## 5 Question 9

Convert the following nfa into equivalent dfa. The dfa is shown in Fig. 5


Figure 5: Convert nfa into equivalent dfa

## 6 Question 10 (b)

Find the minimal dfa's for the following languages:

$$
L=\left\{a^{n} b: n \geq 0\right\} \cup\left\{b^{n} a: n \geq 1\right\}
$$

The first dfa for the language $L$ is shown in Fig. 7 (C). The minimal dfa is depicted in 7 (D). Steps to minimize dfa is shown in Fig. 6

The answer for this problem is found in Fig. 7 D

(a)

| q1 | X |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q2 | X | X |  |  |  |  |
| q3 | X | X | X |  |  |  |
| q4 |  | X | X | X |  |  |
| q5 | X | X | X |  | X |  |
| q6 | X | X | X | X | X | X |
|  | q0 | q1 | q2 | q3 | q4 | q5 |

(b)

| q1 | X |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q2 | X | X |  |  |  |  |
| q3 | X | X | X |  |  |  |
| q4 | X | X | X | X |  |  |
| q5 | X | X | X |  | X |  |
| q6 | X | X | X | X | X | X |
|  | q0 | q1 | q2 | q3 | q4 | q5 |

(c)

| q2 | X | X |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| q3 | X | X | X |  |  |  |
| q4 | X | X | X | X |  |  |
| q5 | X | X | X |  | X |  |
| q6 | X | X | X | X | X | X |
|  | q0 | q1 | q2 | q3 | q4 | q5 |

(d)

Figure 6: Minimal dfa procedures: (a) first scan for all pairs (b-c) repeating steps, (d) final table


Figure 7: (A) $L=a^{n} b$, (B) $L=b^{n} a$, (C) Language L , (D) minimal dfa

