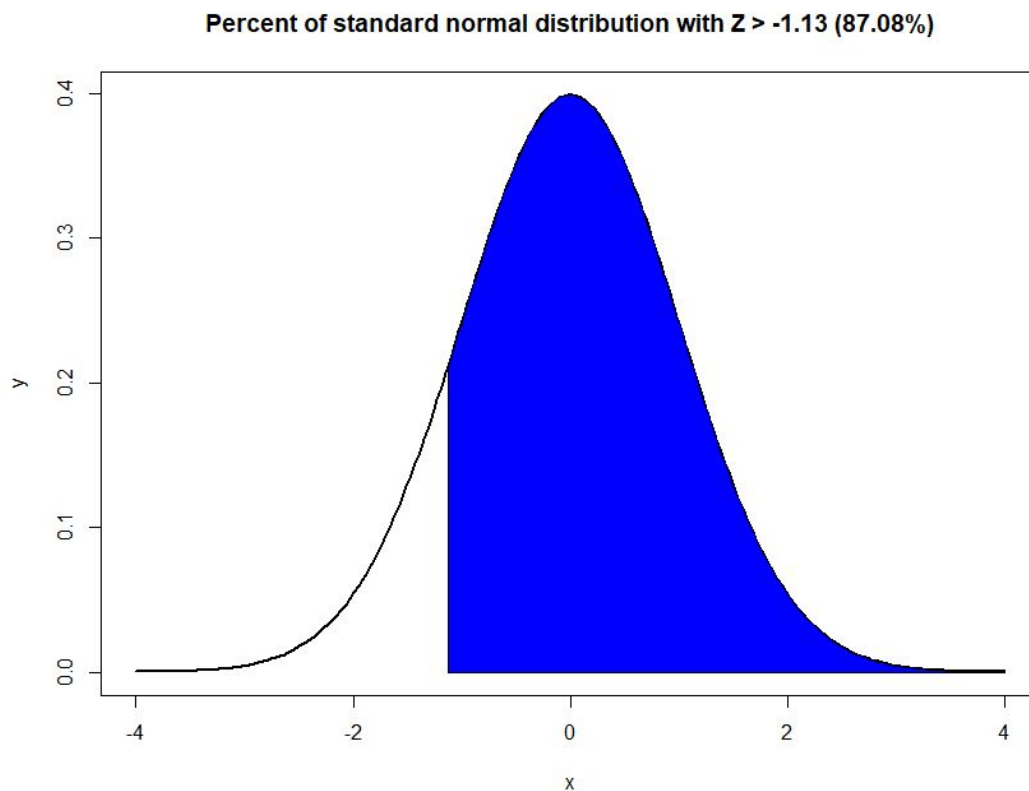


Assignment #6 - Chapter 4

Question 4.2

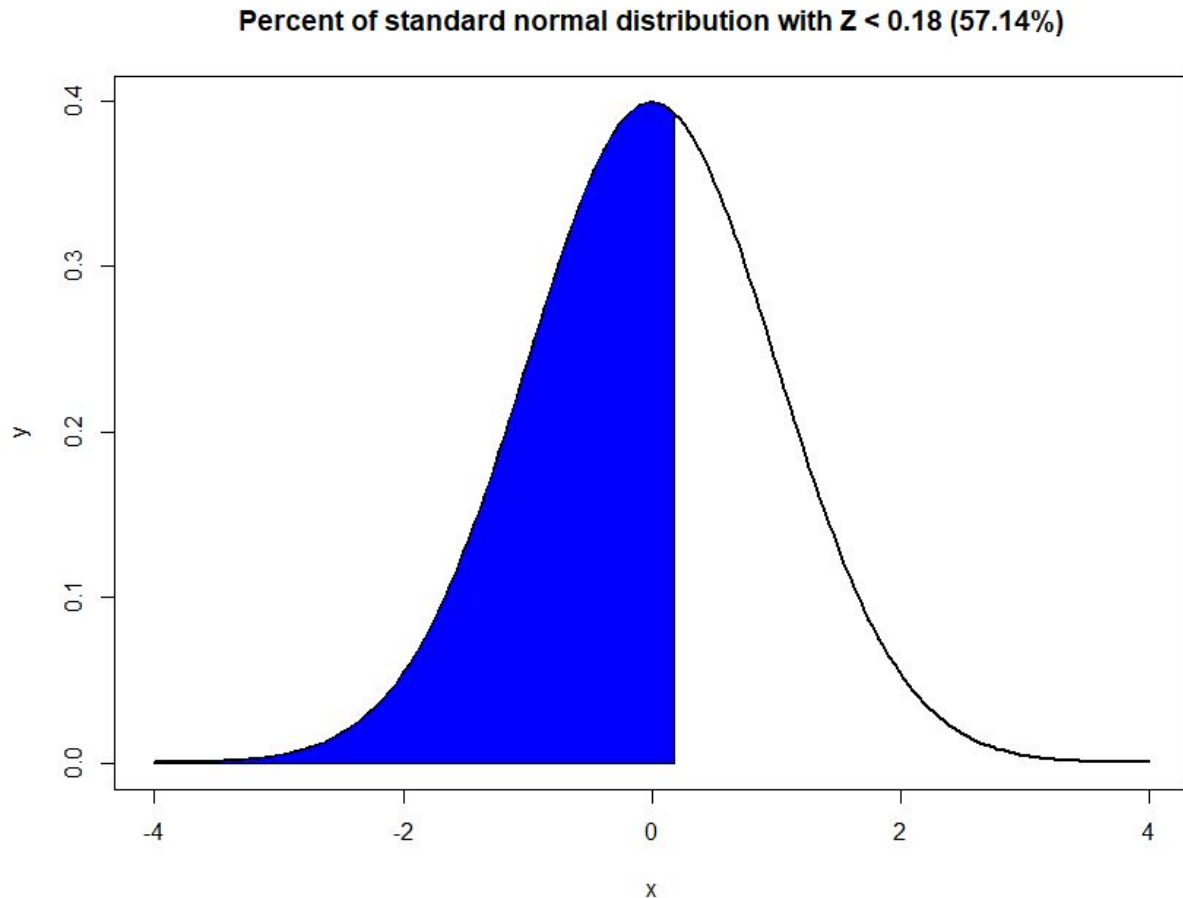
Area under the curve, Part II. What percent of standard normal distribution $N(\mu = 0, \sigma = 1)$ is found in each region? Be sure to draw a graph.

- a. $Z > -1.13$



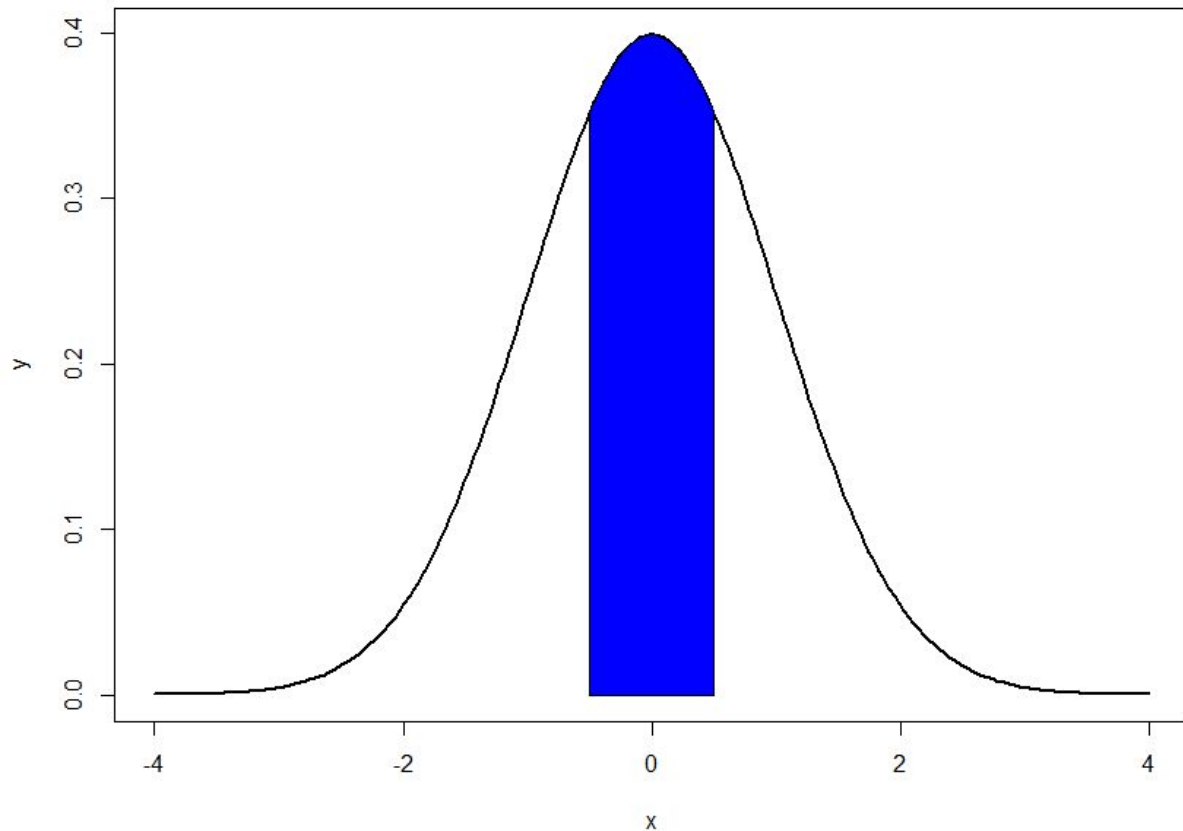
Ans: To find the percent of the standard normal distribution of $Z > -1.13$ we need to subtract the given area from one $\rightarrow 1 - \text{pnorm}(-1.13) = 1 - 0.1292381 = 0.8707619$
(87.08%)

-
- b. $Z < 0.18 \rightarrow$ this is the area under the curve on the left: $\text{pnorm}(0.18) = 0.5714237$
(57.14%)



- c. $Z > 8$. Similar to (a) $\rightarrow 1 - \text{pnorm}(8) = 6.661338e-16$ (**6.66e-14 % = 0%**)
- d. $|Z| < 0.5 \rightarrow -0.5 < Z < 0.5$. First find the percent below -0.5 then the percent above $.5$.
The final answer will be: $1 - \text{percent_below}(-0.5) - \text{percent_above}(0.5) = 1 - \text{pnorm}(0.5) = 1 - \text{percent_below}(-0.5) - (1 - \text{percent_below}(0.5)) = 1 - \text{pnorm}(-0.5) - (1 - \text{pnorm}(0.5)) = 0.3829249$ (**38.29%**)

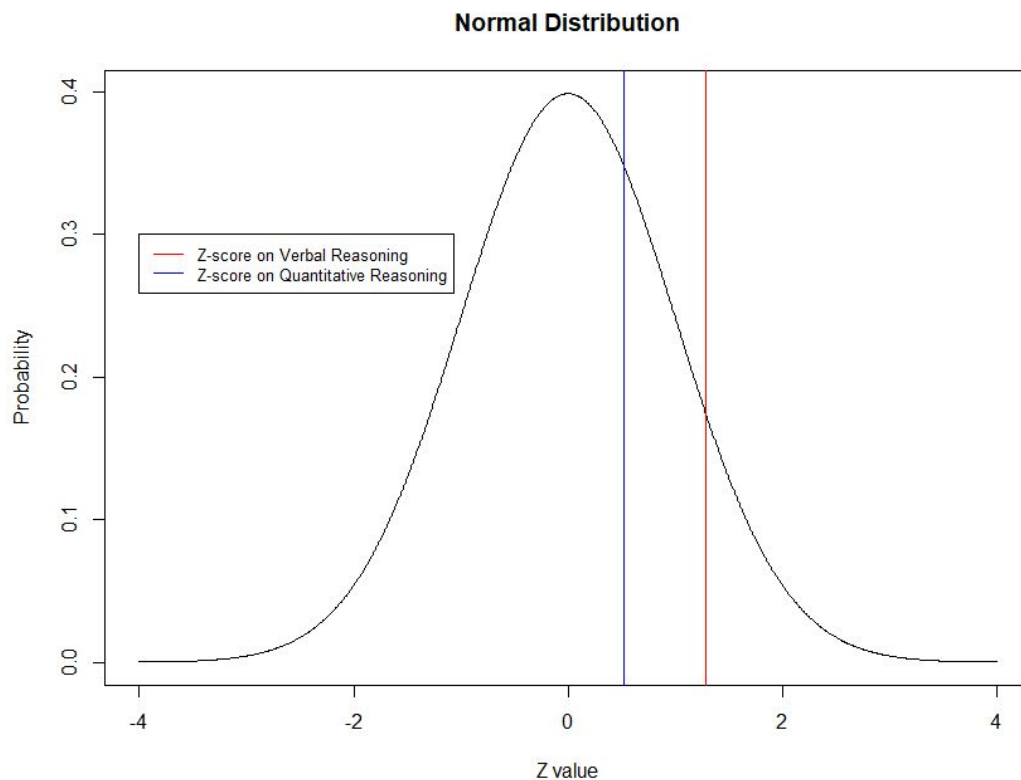
Percent of standard normal distribution with $|Z| < 0.5$ (38.29%)



Question 4.3 GRE Scores, Part I

Sophia who took the Graduate Record Examination (GRE) scored 160 on the Verbal Reasoning section and 157 on the Quantitative Reasoning section. The mean score for the Verbal Reasoning section for all test takers was 151 with a standard deviation of 7, and the mean score for the Quantitative Reasoning was 153 with a standard deviation of 7.67. Suppose that both distributions are nearly normal.

- a. Write down the shorthand for these two normal distributions.
 - Verbal Reasoning section: $N(\mu = 151, \sigma = 7)$
 - Quantitative Reasoning section: $N(\mu = 153, \sigma = 7.67)$
- b. What is Sophia's Z-score on the Verbal Reasoning section? On the Quantitative Reasoning section? Draw a standard normal distribution curve and mark these two Z-scores.



Ans:

- Z-score on the Verbal Reasoning section: $(160-151)/7 = \mathbf{1.2857}$
- Z-score on the Quantitative Reasoning section: $(157-153)/7.67 = \mathbf{0.5215}$

c. *What do these Z-scores tell you?*

Ans: The Z-scores suggest the number of standard deviations they fall above or below the mean, that is, Sophia's scored 1.2857 standard deviations above the mean for the Verbal Reasoning section and 0.5215 standard deviations above the mean for the Quantitative Reasoning section.

d. *Relative to others, which section did she do better on?*

Ans: Sophia performed better on the Verbal Reasoning section than she did on the Quantitative Reasoning section (higher Z-score)

e. *Find her percentile scores for the two exams.*

-
- Percentile scores for Verbal Reasoning: $\text{pnorm}(1.2857) = 0.9007261$ (**90th percentile**)
 - Percentile scores for Quantitative Reasoning: $\text{pnorm}(0.5215) = 0.6989907$ (**70th percentile**)

f. *What percent of the test takers did better than her on the Verbal Reasoning section? On the Quantitative Reasoning section?*

Answer:

- Verbal Reasoning section: $1 - \text{pnorm}(1.2857) = 0.09927389$ (**9.92%**)
 - Quantitative Reasoning section: $1 - \text{pnorm}(0.5215) = 0.3010093$ (**30.1%**)
- g. *Explain why simply comparing raw scores from the two sections could lead to an incorrect conclusion as to which section a student did better on.*

Answer: First it may not use the same scale of measurement. Even if it did use the same scale, these scores are not in the same dimension. There may be a random error in one dimension (for example, system error) that makes her score not reflect the true score. For example, we can't say he performed badly while others also answer the same question wrong (this may be a system error). Only by comparing her score with the population or standardized scores can make sense.

h. *If the distributions of the scores on these exams are not nearly normal, would your answers to parts (b) - (f) change? Explain your reasoning.*

Answer: The only issue that we need to take care of is to use the function pnorm , in another word, to use the area under the curve to calculate a percentage. For this reason, answers for (e) and (f) must change.

Question 4.5 GRE Scores, Part II

- a. *The score of a student who scored in the 80th percentile on the Quantitative Reasoning section.*

Ans: To find X , we need to find Z -score, given the 80th percentile we can find the boundary that determines this area by using $qnorm(.8) = 0.8416212 = Z$

$$\text{Then } X = SD * Z + \text{Mean} = 7.67 * 0.8416212 + 153 = \mathbf{159.4552}$$

- b. *The score of a student who scored worse than 70% of the test-takers in the Verbal Reasoning section.*

Ans: A student who scored worse than 70% of the test takers means that she/he belongs to the first 30%, we have $Z = qnorm(.3) = -0.5244005$

$$\text{Then } X = SD * Z + \text{Mean} = 7 * (-0.5244005) + 151 = \mathbf{147.3292}$$